

Determination of Normal Operating Heights of Reservoirs in a Network Using Non-linear Reservoir Method

Abstract: Utilizing the coupled tank approach to the modeling of the reservoir network for flood control will be incomplete unless data on the reservoirs' normal operating heights are presented. The normal operating heights are derived at the equilibrium points and upon the substitution of these equilibrium points into the Jacobian matrix, we obtained a diagonally dominant tridiagonal matrix with negative diagonal elements. This matrix gives a negative definite matrix which satisfies the desired stability condition. For each rain rate used, surface area of reservoir as well as the normal operating heights that will ensure stability has been computed. In conclusion, this approach provides a satisfaction tool to determine the parameters that will ensure stability within the lowest and highest point in a given catchment.

Keywords: Reservoir; Coupled Tanks; bifurcation analysis; Normalization; tridiagonal matrix; flood control; Steady State.

Introduction

Flood phenomena are a major environmental issue that is confronting both rural and urban settlements on a global scale. In this point of view, there is a need to develop a more sustainable approach to flood management. It can be seen that the storage of water in a reservoir or water transfer between two or more reservoirs is determined by several factors, which include hydrology, hydraulics, topology, vegetation, perviousness and environmental standards [1]. Consequently, modeling of water upon flow, transfer and storage is usually complex due to various interacting factors.

Routing is an analytical procedure that is used to evaluate water flow through hydrological systems with rainfall runoff as the input. There are different forms depending on approach and platform where it is been evaluated. Hydrological routing utilizes the continuity equation to derive the famous Muskingum equation [2]. This is known as channel routing since the activities takes place along the conveyance. The Muskingum-Cunge was then modified for a large drainage network with compound cross sections and continuous long time simulation [2]. Hydraulic routing model takes place in distributed systems and kinematic wave method is used to evaluate routing

in the system. It is also Muskingum-Cunge approach that serves as base model for hydrologic drainage network channel flow routing model which is evaluated in four components, there are drainage network topology, hydraulic properties of cross section, routing the flow in individual channel reaches and routing in the network. Routing in ponds and reservoirs is referred to as flood routing since they are used as storage devices to attenuate flood effect. Level pool of water is a tool on which the continuity equation in terms of height of water in the reservoir is relied. Hence, the name of nonlinear reservoir model is used for flood routing[3].

Table1: Variables and Parameters

Parameters	Description
A_j	Surface area of j^{th} reservoir
A_{scj}	Area of sub-catchment
a_j	Size of orifice (m^2)
$C_{d,j}$	Discharge coefficient of R_j
f	Average infiltration rate (m / s) at time t
g	Acceleration due to gravity (m / s^2)
H_j	Height of the reservoir $R_j (m)$
h_{perm}	Permanent height of the retention pond (m)
h_{temp}	Temporary height of the retention pond (m)
h_{flood}	Flood height of the retention pond (m)
h_j^0	Normal operating height $R_j (m)$
I	Rainfall intensity or inflow (m / s)
$H_{i,j,k}^0$	Normal operating height of reservoir i, j or $k (m)$
n	Manning's roughness value for the surface
Q	Flow or discharges (m^3 / s)
S	Slope of overland surface (%)
t_c	Time of concentration (s)
W	Width of the catchment (m)

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41 Nonlinear reservoir routing method is a famous routing method which has been used for different
42 catchments for flood control[2, 3]. The water level depends on the inflow and outflow of water in
43 the reservoir. The inflow consists of runoff and discharges from upland reservoir, while the
44 outflow is divided as evapotranspiration, infiltration and discharges to reservoir on the lower
45 side. The solution of nonlinear reservoir is obtained through the finite difference method[3]. This
46 solution approach is to monitor water level in the reservoir between two fixed points using the
47 variation in inflow and outflow of water within a time interval. This is a single differential
48 equation with two unknowns and is solved using finite difference method. In some cases the
49 value of one of these two unknown variables is usually assumed. Since a single pond is not
50 capable of controlling or preventing flooding from happening in a catchment, rather it exacerbates
51 its influence[4-6]. It is important to consider a nonlinear reservoir method for a network of ponds
52 so that water flow between the reservoirs can be monitored and used for flood control.

53 Detention and retention ponds are used for flood control, where detention pond stores water
54 temporarily, and retention pond has permanent pool of water. Aside this, retention ponds are
55 reservoir for water treatment and flood control, where they take care of the quality and the
56 quantity of water at a given time. Usually, detention pond is designed to have maximum capacity
57 corresponding to the time of concentration for a return period of 25 years [7-11], while studies
58 have shown that retention basins are successful flood control constructs especially for pervious soil
59 with frequent storm events[4]. In doing this, retention pond must have capacity to store the
60 complete runoff at least to its permanent mark. In [12], post development of peak discharge rate
61 is with return period of 2-100 years and their corresponding predevelopment level is often used
62 as the flood control criterion. Hydrological analysis in respect of the detention pond and the
63 associated probabilistic approach for pond sizing were extensively discussed in [13-
64 16]. Comparison of kinematic wave and nonlinear reservoir method was extensively explored
65 in [17], while a full description of routing methods and their selection criteria for specific use are
66 fully discussed in [5].

67 Based on the design considerations of the detention and retention ponds, the nonlinear reservoir
68 model is suitable for the network of reservoir in flood control. The coupled tank system approach

is usually used in the laboratory to study the control of water between tanks. The inflow and outflow through the orifice are used in designing the controller to study water levels in each tank [18-20]. This approach is then extended to the network of reservoir for flood control using nonlinear reservoir model. Network configuration varies depending on the hydrology, hydraulics, topology, vegetation, perviousness and environmental standard [4]. In this work, a series of reservoir network connection is used to study the stability of flow through the normal operating heights that will ensure free flow of water in the network.

Solution to system of nonlinear ordinary differential equations (ODEs) is problem dependent, where the degree of nonlinearity determines its complexities. To system engineer, linearization is the best option to design the controller for the system. On the other hand, this linearized system must satisfy certain conditions which revolve around the stability of the system. Stability study is thus an aspect where all works on dynamic system converges. Either in finding solution to dynamic systems or in its analysis or better still for design, stability remains a tool that guarantees reliability of the result. The nonlinear reservoir model for network of pond is one of such problems; see for examples [21-23]. In this study, the nonlinearity of the proposed model formulation is critically investigated and used as criteria for analytically determining the stability and the normal operating height for each reservoir in the network.

Model Formulation

This work relies on the coupled tank system approach to model network of ponds for flood control, since a single pond cannot control flood effectively in a catchment. The proposed model formulation is aimed to determine the normal operating heights of reservoirs in a network. To maintain free flow of water, a criterion is set based on the designed parameters of the catchment under study. To establish a foundation for the work the following assumptions are relied upon:

1. All ponds in the network are detention ponds except the last down-most pond is the retention pond.
2. The return period for detention ponds is less than that of retention pond designed.
3. The capacity design of all detention ponds must not exceed the permanent storage of the retention pond, that is,

$$97 \quad \sum_{j=1}^{m-1} V_j + v \leq V(h_j^{perm})$$

98 where v is the initial volume in the retention pond, V_m is the volume of retention pond, while V_1 ,
 99 $V_2, \dots, V_{m-1}$ are the volume of all $(m-1)$ detention ponds in the network. In each reservoir at any
 100 time t the water balance is given by

$$101 \quad V = V_{stored} + V_{flowin} + V_{precipitation} - V_{flowout} - V_{evaporation} - V_{seepage} \quad (1)$$

102 Equation (1) gives a full description of water balance in a reservoir, which has been utilized in
 103 different form to model water balance in a reservoir. Consideration of precipitation, evaporation
 104 and seepage vary considerably between catchments, but insignificant in most cases. Several
 105 routing models have been developed in most cases for a single reservoir in a network which
 106 capture these losses. The nonlinear reservoir routing method utilizes the changes in the height of
 107 water in the reservoir with input and output as runoff and discharges respectively. This is
 108 particularly considered as an acceptable routing technique for storm duration longer than the
 109 watershed time of concentration [3, 17]. From (1), the change in height from conservation law is
 110 expressed as

$$111 \quad A_1 \frac{dH_1}{dt} = A_1 I - Q_{0,1} \quad (2)$$

112 where the discharges are either through an orifice or a weir, that is

$$113 \quad Q_{0,1} = \begin{cases} c_{d,1} a_1 (2gH_1)^{1/2} \\ \frac{WH^{5/3} S^{1/2}}{n} \end{cases}$$

114 Equation (2) can be extended to two reservoirs in a network as given below:

$$115 \quad \left. \begin{aligned} A_1 \frac{dH_1}{dt} &= A_1 I - Q_{0,1} \\ A_2 \frac{dH_2}{dt} &= A_2 I + Q_{0,1} - Q_{0,2} - A_2 f \end{aligned} \right\} \quad (3)$$

where the infiltration term in the second equation is due to the function of reservoir as retention pond. Using either orifice or weir does not change in any way affects the model formulation. Form reservoirs flood control network we have:

$$\left. \begin{aligned} A_j \frac{dH_j}{dt} &= A_{j_{sc}} I + B_i \sqrt{H_i - H_j} - B_j \sqrt{H_j - H_k} \quad ; j = 1, 2, \dots, m-1 \\ A_j \frac{dH_j}{dt} &= A_{j_{sc}} I + B_i \sqrt{H_i - H_j} - \frac{W}{n} H_j^{5/2} S^{1/2} - A_j f \quad ; j = m \end{aligned} \right\} \quad (4)$$

where i 's are the inflows to reference reservoir R_j and k is the outflow from reference reservoir R_j , i, j and k are the connected reservoirs and $B_j = c_{d,j} a_j \sqrt{2g}$. Equation(4) is a system of nonlinear ordinary differential equations which capture the storage, inflow and outflow relationship in the network.

The solution to nonlinear system is generally problem dependent and the degree of nonlinearity determines the complexities. To overcome these, we normalize (4) about the normal operating heights of each reservoir and study the stability of (4) to ensure that the solution to the linearized form can be used as an approximate solution of the nonlinear system. In view of this, we define the normal operating heights as the heights which water is expected to be in each reservoir in order to maintain balance. It is important to note that normal operating heights are equilibrium points on which the stability of nonlinear system can be observed. It can also be interpreted as steady state of heights. This steady-state is defined as

$$\begin{aligned} H_j(s) &= h_j^0 = h_j^{\max} \quad ; \quad V(h_j^{\max}) = V_{\max} \quad \text{and} \quad j = 1, 2, \dots, m-1 \\ H_j(s) &= h_j^0 = h_j^{\text{perm}} \quad ; \quad V(h_j^0) = V(h_j^{\text{perm}}) \quad \text{and} \quad j = m \end{aligned}$$

where h_j^{perm} is the lowest height of retention pond followed by the temporary height h_j^{temp} and flood height h_j^{flood} . From the third assumption,

$$V(h_j^{\text{perm}}) = V(h_1^{\max}) + V(h_2^{\max}) + \dots + V(h_{j-1}^{\max}) + v \quad \text{at } t \leq t_c, \quad \text{where } v \in V_j \quad ; j=m$$

The volumes of the three heights of the retention pond are designed for different return periods[24] to serve the purpose of quantity and quality of water needed for recreation and guide against flood. The normalization of (4) about the normal operating heights is

$$\left. \begin{aligned}
\frac{dh_j}{dt} &= \frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} h_i - \left(\frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} + \frac{B_j}{2A_j\sqrt{H_j^0 - H_k^0}} \right) h_j + \frac{B_j}{2A_j\sqrt{H_j^0 - H_k^0}} h_k \\
j &= 1, 2, \dots, m-1, \\
\frac{dh_j}{dt} &= \frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} h_i - \left(\frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} + \frac{5WS^{1/2}}{3A_jn} (H_j^0)^{2/3} \right) h_j \\
j &= m
\end{aligned} \right\} \quad (5)$$

Where $H_j = h_j + H_j^0$ and $\dot{H}_j = \dot{h}_j$ since H_j^0 is a constant

The normal operating heights are conceptualized as the steady state which will guarantee stability of the system. In this respect, (5) can be expressed in terms of steady state heights by replacing normal operating height H_j^0 with steady state height $H_j(s)$

This is a system of linear equations which gives a linearized solution of nonlinear system in (4) about the steady state heights. Linearization of nonlinear systems may not in general produce a better approximation but, in order to guarantee that the linearized system behaves in the same way as the original nonlinear system, the linearized system must be stable about the steady state heights [25, 26]. This degree of nonlinearity of a system varies between mildly and highly nonlinearities. In practice, linearizing a nonlinear system is a common practice and the result is comparable with the original nonlinear system if bifurcation analysis shows that the system is stable about its steady state [27, 28]. As the solution to (5) approximates (4), the height in each reservoir has to be bounded so that the solution will be tractable. Stability studies can give accurate description of the system without necessarily solving the problem. Since the problem has become constrained system of ODEs i.e. each height must be bounded between the minimum and maximum point. The minimum point is not necessarily zero height but height corresponding to zero volume. The boundedness of heights will ensure that we will not get heights which correspond to negative volume. Studying the behavior of the system through stability is more tractable to determine the condition of free flow of water. For reservoirs $j=1,2,\dots,m$ the system matrix associated with (5) is the Jacobian matrix associated with (4) which will be used as stability decision tool.

Stability Analysis of the Steady State Heights

The stability of the system (4) is studied around the steady state height which is obtained by solving (4) when $\dot{H}_j = 0 \forall j$. Starting with the inflow $i=0$ in (4) for $j=1,2,\dots,m-1$ we have

$$H_j(s) = H_k(s) + \left(\frac{A_{scj} I}{B_j} \right)^2 ; i = 0$$

This serves as the input to all reservoirs which receives inflow from the upper reservoirs, and in general, the steady state height of m connected reservoirs network is

$$\left. \begin{aligned} H_j(s) &= \left(\frac{I}{B_j} \sum_{i=1}^j A_{sci} \right)^2 + H_k(s) ; j = 1, 2, \dots, m-1 \\ H_j(s) &= \left(\frac{n}{WS^{1/2}} \left(I \sum_{i=1}^j A_{sci} - A_j f \right) \right)^{3/5} ; j = m \\ i \neq 0, j &:= i \text{ and } k := j \end{aligned} \right\} (6)$$

Equation (6) shows that there is an inter-relationship between the steady state heights of two adjacent reservoirs from the first reservoir down to the m^{th} reservoir. The full description of the steady state may not have been reflected in (6) since water discharge from the m^{th} reservoir is going to the connecting stream which also has its normal operating height. Incorporating this into (4) we have

$$\left. \begin{aligned} A_j \frac{dH_j}{dt} &= A_{jsc} I + B_i \sqrt{H_i - H_j} - B_j \sqrt{H_j - H_k} ; j = 1, 2, \dots, m-1 \\ A_j \frac{dH_j}{dt} &= A_{jsc} I + B_i \sqrt{H_i - H_j} - \frac{W}{n} (H_j - H_s)^{5/3} S^{1/2} - A_j f ; j = m \end{aligned} \right\} (7)$$

and as in (6), the steady state becomes

$$\left. \begin{aligned} H_j(s) &= \left(\frac{I}{B_j} \sum_{i=1}^j A_{sci} \right)^2 + H_k(s) ; j = 1, 2, \dots, m-1, \\ H_j(s) &= \left(\frac{n}{WS^{1/2}} \left(I \sum_{i=1}^j A_{sci} - A_j f \right) \right)^{3/5} + H_s(s) ; j = m \\ i \neq 0, j &:= i \text{ and } k := j \end{aligned} \right\} (8)$$

Equation (8) shows that the entire normal operating height of the reservoir in the network can be estimated from the normal operating height of the connecting stream or river. Consider dropping of the discharge term in the m^{th} equation and proceeding in the same way we have;

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$$I \sum_{i=1}^j A_{sci} - A_j f = 0 ; j = m \quad (9).$$

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This can simply be interpreted as that the total runoff in the entire catchment is equal to total water loss due to infiltration in the retention pond. The absence of the height in (9) is already an indication that it cannot be normal operating height at the m^{th} reservoir when there is no discharge from it. The steady state heights obtained from (6) can be substituted in (5) to get the coefficient matrix of the system. This matrix can also be generated by

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$$\frac{\partial \dot{H}_j}{\partial H_r} ; \quad r = i, j, k$$

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Bifurcation analysis relies on the Jacobian matrix evaluated at the steady state height to determine the stability of the network. The lumped hydrologic and hydraulic parameters in the Jacobian matrix have influence on the nature of the matrix that will be used for stability study. The number of reservoirs in the network determines the dimension of the matrix while network configuration determines its nature. A series of reservoir connection will produce $m \times m$ non-symmetric tridiagonal diagonally dominant matrix while a hypothetical network will produce $m \times m$ non-symmetric diagonally dominant matrix. Theoretically, the unequal area of the sub-catchment is responsible for the non-symmetric nature of these matrices. Equal areas are, however, rare to come across in practice.

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Let the Jacobian matrix associated with (7) be $H(s)$ and λ_j 's are the corresponding eigenvalues. By Poincare Lyapunov theorem, if $\lambda_j \neq 0 \forall j$, the trajectory of the system around the steady state behaves in the same way as the associated linear systems obtained through the linearization of this nonlinear system [25]. In fact, the Jacobian matrix is the best linear approximation to a continuously differentiable function near a given point [25]. The decision on the nature of stability of the steady state rests on the eigenvalue or the definiteness of the matrix. In this formulation, the steady state obtained will produce negative definite Jacobian matrix which is desired to get global stability for the system.

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By similarity transformation, the non-symmetric tridiagonal matrix can be transformed into a symmetric matrix. This ensures that the eigenvalues are real and are equal to the eigenvalues of the original non-symmetric matrix [29]. This transformation rests on the condition that $a_{j+1,j} / a_{j,j+1} > 0, \forall j \in m$ i.e. none of the subdiagonal must be zero since the matrix symmetric condition is assured. The steady state is hyperbolic if for every j, λ_j are real. For $\lambda_j > 0; \forall j$, the steady state is a source and the system is unstable, whereas if $\lambda_j < 0; \forall j$, the steady state is a sink and the system is stable. But, if λ_j admits both positive and negative signs, then the steady state is a saddle point [26]. In our study, a system in which all the eigenvalues are negative is desired. This will guarantee the free flow of water in the catchment and at the same time keep the water level around the operating heights.

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215 **Proposition**216 *Let A_n be the real symmetric tridiagonal matrix with diagonal entries negative. If*

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$$b_i^2 \leq \frac{1}{4} a_i a_{i+1} \left(1 + \frac{\pi^2}{1 + 4n^2} \right) \text{ for } i=1,2,\dots,n-1, \text{ then the determinant of } A_n \text{ satisfies}$$

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$$\det A_n < 0 ; \forall n$$

219 This is a direct consequence of proposition (2.1) in [29] which consider the positive diagonal
220 entries.221 **Proof:**

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$$\text{Given } A_n = \begin{pmatrix} -a_1 & b_1 & & & \\ b_1 & -a_2 & b_2 & & \\ & b_2 & \ddots & \ddots & \\ & & \ddots & \ddots & b_{n-1} \\ & & & b_{n-1} & -a_n \end{pmatrix}$$

Let $A'_n = -A_n$, then $b_i < 0$, $a_i > 0$ and $a_i a_{i+1} > 0$ since $b_i^2 \leq \frac{1}{4} a_i a_{i+1} \left(1 + \frac{\pi^2}{1 + 4n^2} \right)$ then A'_n is positive definitei.e. $\det A'_n > 0 \forall n$. Therefore A_n is negative definite since $A_n = -A'_n$ □223 Since the non-symmetric nature of the Jacobian matrix can be taken care of by the similarity
224 transformation, the eigenvalues of both matrices are equal. The above proposition shows that the
225 normal operating heights which produce the Jacobian matrix will always make the network
226 stable.

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228 **Example**229 Consider a catchment endowed with 10 sub-catchments which can be connected in series as
230 shown in Figure 1, the hydrologic and hydraulic parameters are presented in table 2. Determine
231 the normal operating heights of each reservoir in the network that will guarantee free flow of
232 water.

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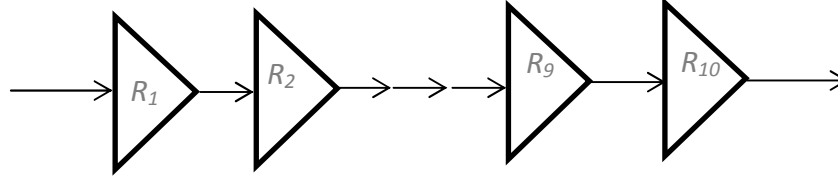


Figure1. A network of 10 reservoirs connected in series

Results and Discussions

The network as shown in Figure1 consists of 10reservoirs; the Jacobian matrix associated with (4) is given by (10). This is a non-symmetric tridiagonal matrix, whose entries are obtained from the hydrologic and hydraulic parameters of each sub-catchment. These parameters are given in Table1. It can be noticed in (10) that the non-symmetric nature is due to the surface area of each reservoir.The similarity transformation will produce a matrix which will retain the diagonally dominant property of the original matrix and at the same time makes it symmetric. These are conditions that guarantee the definiteness of the matrix,

$$H(s) = \begin{pmatrix} -\frac{\delta_1}{A_1} & \frac{\delta_1}{A_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\delta_1}{A_2} & -\left(\frac{\delta_1}{A_2} + \frac{\delta_2}{A_2}\right) & \frac{\delta_2}{A_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\delta_2}{A_3} & -\left(\frac{\delta_2}{A_3} + \frac{\delta_3}{A_3}\right) & \frac{\delta_3}{A_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\delta_3}{A_4} & -\left(\frac{\delta_3}{A_4} + \frac{\delta_4}{A_4}\right) & \frac{\delta_4}{A_4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\delta_4}{A_5} & -\left(\frac{\delta_4}{A_5} + \frac{\delta_5}{A_5}\right) & \frac{\delta_5}{A_5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\delta_5}{A_6} & -\left(\frac{\delta_5}{A_6} + \frac{\delta_6}{A_6}\right) & \frac{\delta_6}{A_6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\delta_6}{A_7} & -\left(\frac{\delta_6}{A_7} + \frac{\delta_7}{A_7}\right) & \frac{\delta_7}{A_7} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\delta_7}{A_8} & -\left(\frac{\delta_7}{A_8} + \frac{\delta_8}{A_8}\right) & \frac{\delta_8}{A_8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\delta_8}{A_9} & -\left(\frac{\delta_8}{A_9} + \frac{\delta_9}{A_9}\right) & \frac{\delta_9}{A_9} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\delta_9}{A_{10}} & -\left(\frac{\delta_9}{A_{10}} + \frac{\delta_{10}}{A_{10}}\right) \end{pmatrix} \quad (10)$$

with

$$\delta_j = \frac{B_j^2}{2I \sum_{i=1}^j A_{sc_i}}; \quad j = 1, 2, \dots, m-1$$

$$\delta_j = \frac{5}{3} \left(\frac{WS^{1/2}}{n} \right)^{3/5} \left(I \sum_{i=1}^j A_{sc_i} - A_j f \right)^{2/5}; \quad j = m$$

$$\text{and } A_j = \left(\frac{A_{sc_j} I_{sc_j} t_c}{k} \right)^{1/\tau}; \quad A_j < A_{sc_j}$$

where the relationship between the area of catchment and area of reservoir was established in [30] and the value of k and τ have been respectively found to be $1/120$ and $3/2$ in [31].

We desire negative eigenvalues or negative definiteness as criteria that guarantee free flow or stability of the system. It was observed in [29] that the general characterization of positive definiteness, which is given by eigenvalues, seems inadequate as far as the applied and numerical matrix theories are concerned. This is because of the enormous computational efforts that are involved in the calculations of the eigenvalues. Depending on the magnitude of the problem and availability of computational tools, the principal minors due to Sylvester's theorem or the eigenvalue can be used as decision criteria.

From the results presented in Table 2, it is observed that the rain intensities used together with the time of concentration of the sub-catchment determine the surface area of the reservoir. Either the reservoirs are regular or irregular, pyramidal consideration was used in [31] to derive the area-volume relationship of reservoir in a catchment. This relationship is used in [30] to determine the surface area of reservoir from area of catchment. For each rain rate, the surface area of reservoir the associated normal operating height is determined for each sub-catchment.

In an existing network, the data on the normal operating height is substituted into the Jacobian matrix and the stability is determined. If the stability test fails, there are a number of reasons that may cause it from happened. The most important of all is the sediment accumulation in the reservoirs which has caused reduction in the effective capacity of the reservoirs. Determination of minimum sediment that causes this instability is the question that requires attention and ability to redesign the flow pattern and hydraulic selection can restore the desired stability.

Table 2: Computed Normal operating Heights and respective eigenvalues

j	c_{dj}	Asc_j	a_j	t_c	A_j			$H_j(s)$			λ_j		
					$Min(I)$	$Average(I)$	$Max(I)$	$Min(I)$	$Average(I)$	$Max(I)$	$Min(I)$	$Average(I)$	$Max(I)$
1	0.33	2200	10	11.1315	110.8049	338.991	594.4526	0.5677	3.1625	11.0108	-245.5648	-15.0204	-3.6886
2	3.32	3100	15	19.19796	200.3954	612.7479	1074.511	0.5641	3.0602	10.4592	-157.628	-9.6416	-2.3677
3	0.62	600	20	17.95822	64.1337	196.1012	343.8819	0.5640	3.0576	10.4452	-56.0947	-3.4311	-0.8426
4	0.33	1850	25	20.66612	149.1982	456.2026	799.994	0.5622	3.0055	10.1642	-7.7837	-0.4761	-0.1169
5	3.32	4600	25	19.03274	259.2085	792.5805	1389.864	0.5551	2.8024	9.0691	-3.9435	-0.2412	-0.001
6	0.62	1630	25	20.12942	134.7377	411.9867	722.4573	0.5549	2.7973	9.0416	-0.0104	-0.0032	-0.0059
7	0.33	1780	30	31.87293	194.1055	593.5153	1040.785	0.5484	2.6101	8.0320	-0.1568	-0.0162	-0.0115
8	3.32	7400	35	33.99316	523.8722	1601.841	2808.979	0.5280	2.0269	4.8870	-0.7157	-0.0451	-0.0243
9	0.62	8140	40	29.99972	513.6149	1570.477	2753.979	0.5276	2.0178	4.8377	-1.6293	-0.1194	-0.0593
10	0.33	9000	50	27.00018	511.9364	1565.345	2744.979	0.5148	1.6512	2.8609	-1.4792	-0.0986	-0.0509

$g=9.8ms^{-2}$, $W=10m$, $S=0.5$, $f=0.013mms^{-1}$, $n=0.25$, $k=1/120$ and $\tau=3/2$ $Min(I)=0.0003972ms^{-1}$, $Ave(I)=0.0021239ms^{-1}$ $max(I)=0.0049319ms^{-1}$

Conclusions

The result presented in Table 2 shows that for each rain rate used the surface area of the reservoirs and the normal operating height ensure stability from the area of sub-catchment at time of concentration. Network configuration hangs on the vegetation and topology while stability

depends on normal operating height obtained from catchment parameters. This network remains stable as long as these parameters are derived from the catchment and satisfy the range of height from the highest down to the lowest. The network configuration which produce diagonally dominant tridiagonal matrix together with the similarity transformation is a great feature that makes computation and analysis of this network interesting.

Retention pond is the most important reservoir in the network because it regulates water flow in the network while at the same time controls the discharges to the connecting stream. Due to this, the effect of the connecting stream on the stability of the network has been shown in (7) and the steady state in (8) consider the normal operating height of the connecting stream on the stability. To effectively capture these behaviors, the network has to be examined at

$$\begin{aligned} h_j &= 0; h_j = h_j^{\max} \text{ as } t \rightarrow \infty; j = 1, 2, \dots, m-1 \\ \text{but, } \dot{h}_j &\neq 0 \text{ at } j = m \text{ i.e. } h_j \in (h_j^{\text{perm}}, h_j^{\text{flood}}) \\ \text{and } \dot{h}_j &\neq 0 \text{ at } j = \text{stream i.e. } h_j \in (h_j^{\min}, h_j^{\max}) \end{aligned}$$

This suggests that we will have two coupled equations derive from $m+1$ equations from where we can determine the time, $h_j = h_j^{\text{flood}}$ and $h_j = h_j^{\max}$. This is the time flood which is expected in the catchment. Aside this, sediment accumulation plays a major role in destabilizing the network since it reduces the effective capacity of the reservoir there by reducing the time estimate for flood occurrence. The next focus will be to address these issues in order to have a complete model description and stability study. This will set the pace for a controller design for the network and at the same time provides a foundation to define optimal control policy for the network.

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